

हमारा विश्वास... हर एक विद्यार्थी है खास

**JEE
MAIN
Sept.
2020**

QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 3 Sep. _ SHIFT - 2



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Q.1 If $x^3 dy + xy dx = x^2 dy + 2y dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to:

- (1) $\frac{\sqrt{e}}{2}$ (2) $\frac{3}{2}\sqrt{e}$ (3) $\frac{1}{2} + \sqrt{e}$ (4) $\frac{3}{2} + \sqrt{e}$

Sol. 2

$$(x^3 - x^2)dy = (2 - x) y dx$$

$$\int \frac{dy}{y} = \int \frac{2 - x}{x^2(x - 1)} dx$$

$$\int \frac{dy}{y} = -\int \frac{x - 1 - 1}{x^2(x - 1)} dx$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x^2} = \int \frac{x^2 - 1 - x^2}{x^2(x - 1)}$$

$$= \frac{1}{x} - \int \frac{x + 1}{x^2} dx + \int \frac{dx}{x - 1}$$

$$\ln|y| = \frac{2}{x} - \ln|x| + \ln|x - 1| + c$$

$$x = 2, y = e$$

$$1 = 1 - \ln 2 + c \Rightarrow c = \ln 2$$

$$\ln|y| = \frac{2}{x} - \ln|x| + \ln|x - 1| + \ln 2$$

$$\text{put } x = 4$$

$$\ln|y| = \frac{1}{2} - 2\ln 2 + \ln 3 + \ln 2$$

$$\ln y = \ln\left(\frac{3}{2}\right) + \frac{1}{2}$$

$$y = \frac{3}{2} \cdot e^{\frac{1}{2}} = \frac{3}{2}\sqrt{e}$$

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Q.2 Let A be a 3×3 matrix such that $\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and $B = \text{adj}(\text{adj } A)$.

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to:

- (1) $\left(9, \frac{1}{81}\right)$ (2) $\left(9, \frac{1}{9}\right)$ (3) $\left(3, \frac{1}{81}\right)$ (4) (3, 81)

Sol. 3

$$\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix} \Rightarrow |\text{adj } A| = 9$$

$$\Rightarrow |A|^2 = 9 \Rightarrow |A| = 3 = |\lambda|$$

$$B = \text{adj}(\text{adj } A) = |A| \cdot A = 3A$$

$$|(B^T)^{-1}| = \frac{1}{|B^T|} = \frac{1}{|B|} = \frac{1}{|3A|} = \frac{1}{27 \times 3} = \frac{1}{81} = \mu$$

$$|\lambda|, \mu = \left(3, \frac{1}{81}\right)$$

Q.3 Let $a, b, c \in \mathbb{R}$ be such that $a^2 + b^2 + c^2 = 1$, If $a \cos \theta = b \cos\left(\theta + \frac{2\pi}{3}\right) = c \cos\left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is

- (1) $\frac{\pi}{2}$ (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{9}$ (4) 0

Sol. 1

$$\cos \alpha = \frac{ab + bc + ca}{a^2 + b^2 + c^2}$$

$$a \cos \theta = b \cos\left(\theta + \frac{2\pi}{3}\right) = c \cos\left(\theta + \frac{4\pi}{3}\right) = \lambda$$

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$$\frac{1}{a} = \frac{\cos \theta}{\lambda}, \frac{1}{b} = \frac{\cos\left(\theta + 2\frac{\pi}{3}\right)}{\lambda}, \frac{1}{c} = \frac{\cos\left(\theta + \frac{4\pi}{3}\right)}{\lambda}$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{\lambda} \left[\cos \theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right) \right]$$

$$= \frac{1}{\lambda} \frac{\sin\left[3\left(\frac{\pi}{3}\right)\right]}{\sin\left(\frac{\pi}{3}\right)} \cdot \cos\left[\frac{\theta + \theta + \frac{4\pi}{3}}{2}\right]$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

$$\sum ab = 0$$

$$\cos \alpha = 0$$

$$\alpha = \frac{\pi}{2}$$

Q.4 Suppose $f(x)$ is a polynomial of degree four, having critical points at $-1, 0, 1$. If $T = \{x \in \mathbb{R} \mid f(x) = f(0)\}$, then the sum of squares of all the elements of T is:

(1) 6

(2) 2

(3) 8

(4) 4

Sol. 4

$$f'(x) = k(x+1)x(x-1)$$

$$f'(x) = k[x^3 - x]$$

Integrating both sides

$$f(x) = k \left[\frac{x^4}{4} - \frac{x^2}{2} \right] + C$$

$$f(0) = C$$

$$f(x) = f(0) \Rightarrow k \left(\frac{x^4}{4} - \frac{x^2}{2} \right) + C = C$$

$$\Rightarrow k \frac{x^2}{4} (x^2 - 2) = 0$$

$$\Rightarrow x = 0, \pm \sqrt{2}$$

$$\text{sum of all of squares of elements} = 0^2 + (\sqrt{2})^2 + (-\sqrt{2})^2 = 4$$

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Q.5 If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$ is $\frac{k}{6}$, then k is equal to:

- (1) $2\sqrt{3} + \pi$ (2) $3\sqrt{2} + \pi$ (3) $3\sqrt{2} - \pi$ (4) $2\sqrt{3} - \pi$

Sol. 4

$$\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$$

$$x = \sin\theta$$

$$\int_0^{\pi/6} \frac{\sin^2\theta}{\cos^3\theta} \cdot \cos\theta d\theta$$

$$\int_0^{\pi/6} \tan^2\theta d\theta = [\tan\theta - \theta]_0^{\pi/6}$$

$$\Rightarrow \left(\frac{1}{\sqrt{3}} - \frac{\pi}{6} \right) = \frac{k}{6}$$

$$\frac{2\sqrt{3} - \pi}{6} = \frac{k}{6}$$

$$k = 2\sqrt{3} - \pi$$

Q.6 If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18 k is equal to:

- (1) 5 (2) 9 (3) 7 (4) 11

Sol. 3

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(\frac{-1}{3x}\right)^r$$

$$= {}^9C_r \frac{3^{9-2r}}{2^{9-r}} (-1)^r \cdot x^{18-3r}$$

$$18 - 3r = 0$$

$$\Rightarrow r=6$$

$$= {}^9C_r \left(\frac{3^{-3}}{2^3}\right) = k$$

$$= \frac{7}{18} = k \Rightarrow 18k = 7$$

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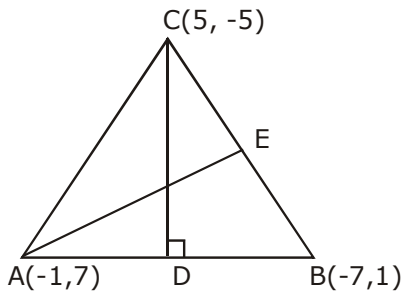
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7. If a ΔABC has vertices $A(-1,7)$, $B(-7,1)$ and $C(5,-5)$, then its orthocentre has coordinates:

- (1) $(-3,3)$ (2) $\left(-\frac{3}{5}, \frac{3}{5}\right)$ (3) $\left(\frac{3}{5}, -\frac{3}{5}\right)$ (4) $(3,-3)$

Sol. 1



equation of CD
 $y + 5 = -1(x - 5)$
 $x + y = 0$ (1)
 equation of AE
 $y - 7 = 2(x + 1)$
 $2x - y = -9$ (2)
 from (1) & (2)
 $x = -3, y = 3$
 Orthocentre = $(-3, 3)$

Q.8. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ ($b < 5$) and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1 e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to:

- (1) $(8,12)$ (2) $\left(\frac{24}{5}, 10\right)$ (3) $\left(\frac{20}{3}, 12\right)$ (4) $(8,10)$

Sol. 4

$$\left. \begin{aligned} \alpha &= 10e_1 \\ \beta &= 8e_2 \end{aligned} \right\} \begin{aligned} b^2 &= 25(1 - e_1^2) \\ b^2 &= 16(e_2^2 - 1) \end{aligned}$$

$$(e_1 e_2)^2 = 1$$

$$\left(1 - \frac{b^2}{25}\right) \left(1 + \frac{b^2}{16}\right) = 1$$

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$$\Rightarrow 1 + \frac{b^2}{25} - \frac{b^2}{25} - \frac{b^4}{400} = 1$$

$$\Rightarrow \frac{9}{16.25} b^2 = \frac{b^4}{400} \Rightarrow b^2 = 9$$

$$\left. \begin{array}{l} e_1 = \frac{4}{5} \\ e_2 = \frac{5}{4} \end{array} \right\} = \left. \begin{array}{l} \alpha = 2ae_1 = 10 \times \frac{4}{5} = 8 \\ \beta = 2ae_2 = 8 \times \frac{5}{4} = 10 \end{array} \right\} = (\alpha, \beta) = (8, 10)$$

Q.9 If z_1, z_2 are complex numbers such that $\operatorname{Re}(z_1) = |z_1 - 1|$, $\operatorname{Re}(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\operatorname{Im}(z_1 + z_2)$ is equal to:

- (1) $2\sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{\sqrt{3}}{2}$

Sol. 1

$$z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$$

$$x_1^2 = (x_1 - 1)^2 + y_1^2 \quad \dots(1)$$

$$\Rightarrow y_1^2 - 2x_1 + 1 = 0$$

$$x_2^2 = (x_2 - 1)^2 + y_2^2 \quad \dots(2)$$

$$y_2^2 - 2x_2 - 1 = 0$$

from equation (1) - (2)

$$(y_1^2 - y_2^2) + 2(x_2 - x_1) = 0$$

$$(y_1 + y_2)(y_1 - y_2) = 2(x_1 - x_2)$$

$$y_1 + y_2 = 2 \left(\frac{x_1 - x_2}{y_1 - y_2} \right)$$

$$\arg(z_1 - z_2) = \frac{\pi}{6}$$

$$\tan^{-1} \left(\frac{y_1 - y_2}{x_1 - x_2} \right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}}$$

$$\therefore y_1 + y_2 = 2\sqrt{3}$$

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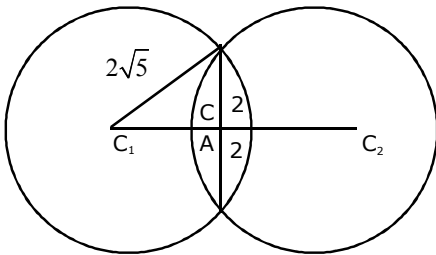
- Q.10** The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval $(0,1)$ is:
 (1) $(-3,-1)$ (2) $(2,4]$ (3) $(1,3]$ (4) $(0,2)$

Sol. 3

$$\begin{aligned} f(0) f(1) &\leq 0 \\ \Rightarrow (2) [\lambda^2 - 4\lambda + 3] &\leq 0 \\ (\lambda - 1) (\lambda - 3) &\leq 0 \\ \Rightarrow \lambda &\in [1, 3] \\ \text{at } \lambda &= 1 \\ 2x^2 - 4x + 2 &= 0 \\ \Rightarrow (x - 1)^2 &= 0 \\ x &= 1, 1 \\ \therefore \lambda &\in (1, 3] \end{aligned}$$

- Q.11** Let the latus rectum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:
 (1) 8 (2) $8\sqrt{5}$ (3) $4\sqrt{5}$ (4) 12

Sol. 1



$$\begin{aligned} C_1 C_2 &= 2C_1 A \\ (C_1 A)^2 + 4 &= (2\sqrt{5})^2 \\ C_1 A &= 4 \\ C_1 C_2 &= 8 \end{aligned}$$

- Q.12** The plane which bisects the line joining the points $(4, -2, 3)$ and $(2, 4, -1)$ at right angles also passes through the point:
 (1) $(0, -1, 1)$ (2) $(4, 0, 1)$ (3) $(4, 0, -1)$ (4) $(0, 1, -1)$

Sol. 3

$$\begin{aligned} &A \text{---} (3, 1, 1) \text{---} B \\ &(y, -2, -3) \quad (3, 1, 1) \quad (2, 4, -1) \\ a &= 2, b = -6 \\ c &= 4 \\ \text{equation of plane} \\ 2(x - 3) + (-6)(y - 1) + 4(z - 1) &= 0 \end{aligned}$$

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$\Rightarrow 2x - 6y + 4z = 4$
passes through (4, 0, -1)

Q.13 $\lim_{x \rightarrow a} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}}$ ($a \neq 0$) is equal to :

- (1) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$ (3) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

Sol. 3
Apply L-H Rule

$$\lim_{x \rightarrow a} \frac{\frac{2}{3}(a+2x)^{-\frac{2}{3}} - 3^{\frac{1}{3}} \cdot \frac{1}{3} x^{-\frac{2}{3}}}{\frac{1}{3}(3a+x)^{-\frac{2}{3}} - 4^{\frac{1}{3}} \cdot \frac{1}{3} x^{-\frac{2}{3}}}$$

$$\Rightarrow \frac{\frac{2}{3}(3a)^{-\frac{2}{3}} - \frac{1}{3^{\frac{2}{3}}} \cdot \left(a^{-\frac{2}{3}}\right)}{\frac{1}{3}(4a)^{-\frac{2}{3}} - \frac{1}{3} \cdot 4^{\frac{1}{3}} \left(a^{-\frac{2}{3}}\right)}$$

$$= \frac{2}{3} \cdot \left(\frac{2}{9}\right)^{\frac{1}{3}}$$

Q.14 Let x_i ($1 \leq i \leq 10$) be ten observations of a random variable X. If $\sum_{i=1}^{10} (x_i - p) = 3$ and $\sum_{i=1}^{10} (x_i - p)^2 = 9$

where $0 \neq p \in \mathbb{R}$, then the standard deviation of these observations is :

- (1) $\frac{7}{10}$ (2) $\frac{9}{10}$ (3) $\sqrt{\frac{3}{5}}$ (4) $\frac{4}{5}$

Sol. 2
Standard deviation
is free from shifting
of origin

$$S.D = \sqrt{\text{variance}}$$

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$$= \sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2}$$

$$= \sqrt{\frac{9}{10} - \frac{9}{100}}$$

$$= \sqrt{\frac{81}{100}} = \frac{9}{10}$$

Q.15 The probability that a randomly chosen 5-digit number is made from exactly two digits is :

- (1) $\frac{134}{10^4}$ (2) $\frac{121}{10^4}$ (3) $\frac{135}{10^4}$ (4) $\frac{150}{10^4}$

Sol. 3

Total case = $9(10^4)$
 fav. case = ${}^9C_2 (2^5 - 2) + {}^9C_1 (2^4 - 1)$
 $= 1080 + 135 = 1215$

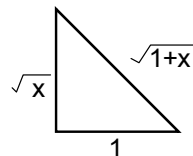
Prob = $\frac{1215}{9 \times 10^4} = \frac{135}{10^4}$

Q.16 If $\int \sin^{-1} \left(\sqrt{\frac{x}{1+x}} \right) dx = A(x) \tan^{-1}(\sqrt{x}) + B(x) + C$, where C is a constant of integration, then the ordered pair (A(x), B(x)) can be:

- (1) $(x+1, -\sqrt{x})$ (2) $(x-1, -\sqrt{x})$ (3) $(x+1, \sqrt{x})$ (4) $(x-1, \sqrt{x})$

Sol. 1

$$\int \sin^{-1} \sqrt{\frac{x}{1+x}} dx$$



$$\int \tan^{-1} \sqrt{x} \cdot \frac{1}{\sqrt{1+x}} dx$$

$$(\tan^{-1} \sqrt{x}) \cdot x - \int \frac{x}{1+x} \cdot \frac{1}{2\sqrt{x}} dx$$

put $x = t^2 \Rightarrow dx = 2t dt$

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$$\begin{aligned}
 &= x \tan^{-1} \sqrt{x} - \int \frac{(t^2)(2tdt)}{(1+t^2)(2t)} \\
 &= x \tan^{-1} \sqrt{x} - t + \tan^{-1} t + c \\
 &= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + c \\
 A(x) &= x + 1, B(x) = -\sqrt{x}
 \end{aligned}$$

Q.17 If the sum of the series $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$ upto n^{th} term is 488 and the n^{th} term is negative, then:

- (1) $n=60$ (2) $n=41$ (3) n^{th} term is -4 (4) n^{th} term is $-4\frac{2}{5}$

Sol. 3

$$20 + \frac{98}{5} + \frac{96}{5} + \dots$$

$$S_n = 488$$

$$\Rightarrow \frac{n}{2} \left[2 \times 20 + (n-1) \left(\frac{-2}{5} \right) \right] = 488$$

$$\Rightarrow 20n - \frac{n^2}{5} + \frac{n}{5} = 488$$

$$\Rightarrow 100n - n^2 + n = 2440$$

$$= n^2 - 101n + 2440 = 0$$

$$\Rightarrow n = 61 \text{ or } 40$$

$$\text{for } n = 40, T_n = 20 + 39 \left(\frac{-2}{5} \right) = +ve$$

$$n = 61, T_n = 20 + 60 \left(\frac{-2}{5} \right) = 20 - 24 = -4$$

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Q.18 Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim p \vee r)$ is F. Then the truth values of p, q, r are respectively :

- (1) F, T, F (2) T, F, T (3) T, T, F (4) T, T, T

Sol. 3

$$(p \wedge q) \rightarrow (\sim q \vee r)$$

Possible when

$$p \wedge q \rightarrow T$$

$$\sim q \vee r \rightarrow F$$

$$\left. \begin{array}{l} p \rightarrow T \\ q \rightarrow T \\ r \rightarrow F \end{array} \right\}$$

$$p \wedge q \Rightarrow T$$

$$\sim q \vee r \rightarrow F \vee F \Rightarrow F$$

$$T \rightarrow F \Rightarrow F$$

Q.19 If the surface area of a cube is increasing at a rate of $3.6 \text{ cm}^2/\text{sec}$, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10cm , is :

- (1) 9 (2) 10 (3) 18 (4) 20

Sol. 1

$$A = 6a^2$$

$a \rightarrow$ side of cube

$$\frac{dA}{dt} = 6 \left(2a \frac{da}{dt} \right) \Rightarrow 3.6 = 12 \times 10 \frac{da}{dt} \Rightarrow \frac{da}{dt} = \frac{3}{100}$$

$$v = a^3$$

$$\frac{dV}{dt} = 3a^2 \frac{da}{dt}$$

$$= 3 \times 100 \times \frac{3}{100}$$

$$= 9 \text{ cm}^3 / \text{sec}$$

Q.20 Let R_1 and R_2 be two relations defined as follows:

$$R_1 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \in \mathbb{Q}\} \text{ and}$$

$$R_2 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \notin \mathbb{Q}\}, \text{ where } \mathbb{Q} \text{ is the set of all rational numbers. Then :}$$

- (1) R_1 is transitive but R_2 is not transitive
 (2) R_1 and R_2 are both transitive
 (3) R_2 is transitive but R_1 is not transitive
 (4) Neither R_1 nor R_2 is transitive


Sol. 4

for R_1

$$\text{Let } a = 1 + \sqrt{2}, b = 1 - \sqrt{2}, c = \frac{1}{8^4}$$

$$aR_1b \quad a^2 + b^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in \mathbb{Q}$$

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$$bR_1c \quad b^2 + c^2 = (1 - \sqrt{2})^2 + \left(8^{\frac{1}{4}}\right)^2 = 3 \in Q$$

$$aR_1c \Rightarrow a^2 + c^2 = (1 + \sqrt{2})^2 + (8^{1/4})^2 = 3 + 4\sqrt{2} \notin Q$$

R_1 is not transitive

R_2

$$\text{let } a = 1 + \sqrt{2}, b = \sqrt{2}, c = 1 - \sqrt{2}$$

$$aR_2b \quad a^2 + b^2 = 5 + 2\sqrt{2} \notin Q$$

$$bR_2c \quad b^2 + c^2 = 5 - 2\sqrt{2} \notin Q$$

$$aR_2c \quad a^2 + c^2 = 6 \in Q$$

R_2 is not transitive

Q.21 If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to ___

Sol. 39

3,, 243
m A.M.

3,, 243
3 G.M.

$$d = \frac{b - a}{n + 1} = \frac{243 - 3}{m + 1} = \frac{240}{m + 1}$$

$$243 = 3(r)^4$$

$$4^{\text{th}} \text{ A.M} = 3 + 4d = 3 + 4\left(\frac{240}{m + 1}\right)$$

$$r = 3$$

$$3 + \frac{960}{m + 1} = 27$$

$$2^{\text{nd}} \text{ G.M.} = ar^2 = 27$$

$$= \frac{960}{m + 1} = 24$$

$$\Rightarrow m = 39$$

Q.22 Let a plane P contain two lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in \mathbb{R}$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in \mathbb{R}$. If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point $M(1, 0, 1)$ to P , then $3(\alpha + \beta + \gamma)$ equals ____

Sol. 5

$$\left. \begin{aligned} \vec{r} &= \hat{i} + \lambda(\hat{i} + \hat{j}) \\ \vec{r} &= -\hat{j} + \mu(\hat{j} - \hat{k}) \end{aligned} \right\}$$

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$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= (-1, 1, 1)$$

equation of plane

$$-1(x - 1) + 1(y - 0) + 1(z - 0) = 0$$

$$\Rightarrow x - y - z - 1 = 0$$

foot of $\perp r$ from $m(1, 0, 1)$

$$\frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = -\frac{(1-0-1-1)}{3}$$

$$x-1 = \frac{1}{3} \quad \left| \frac{y}{-1} = \frac{1}{3} \right| = \frac{z-1}{-1} = \frac{1}{3}$$

$$x = \frac{4}{3}, y = -\frac{1}{3}, z = \frac{2}{3}$$

$$\Rightarrow \left. \begin{array}{l} \alpha = \frac{4}{3} \\ \beta = -\frac{1}{3} \\ \gamma = \frac{2}{3} \end{array} \right\}$$

$$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$$

$$3(\alpha + \beta + \gamma) = 5$$

Q.23 Let S be the set of all integer solutions, (x, y, z) , of the system of equations

$$x - 2y + 5z = 0$$

$$-2x + 4y + z = 0$$

$$-7x + 14y + 9z = 0$$

such that $15 \leq x^2 + y^2 + z^2 \leq 150$. Then, the number of elements in the set S is equal to ____

Sol. 8

$$x - 2y + 5z = 0 \quad \dots(1)$$

$$-2x + 4y + z = 0 \quad \dots(2)$$

$$-7x + 14y + 9z = 0 \quad \dots(3)$$

$$2 \cdot (1) + (2) \text{ we get } z = 0, x = 2y$$

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$$15 \leq 4y^2 + y^2 \leq 150$$

$$\Rightarrow 3 \leq y^2 \leq 30$$

$$y \in [-\sqrt{30}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{30}]$$

$$y = \pm 2, \pm 3, \pm 4, \pm 5$$

no. of integer's in S is 8

Q.24 The total number of 3-digit numbers, whose sum of digits is 10, is _____

Sol. **54**

Let xyz be 3 digit number

$$x + y + z = 10 \text{ where } x \geq 1, y \geq 0, z \geq 0$$

$$\Rightarrow t + y + z = 9$$

$$\left. \begin{array}{l} x - 1 \geq 0 \\ t \geq 0 \end{array} \right\} x - 1 = t$$

$${}^{9+3-1}C_{3-1} = {}^{11}C_2 = 55$$

but for $t = 9, x = 10$ not possible

$$\text{total numbers} = 55 - 1 = 54$$

Q.25 If the tangent to the curve, $y=e^x$ at a point (c, e^c) and the normal to the parabola, $y^2=4x$ at the point $(1,2)$ intersect at the same point on the x-axis, then the value of c is _____

Sol. **4**

$$\text{Tangent at } (c, e^c) \quad y - e^c = e^c (x - c) \quad \dots(1)$$

$$\text{normal to parabola } y - 2 = -1 (x - 1)$$

$$x + y = 3$$

$$\text{at x-axis } y = 0$$

$$\text{in (1), } x = c - 1$$

$$c - 1 = 3 \Rightarrow c = 4$$

$$\dots(2)$$

$$\text{at x-axis } y = 0$$

$$\text{in (2), } x = 3$$

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